	THE ACCUMULATION FUNCTION AND ITS DERIVATIVE (FTC 2)

IN-CLASS SAMPLE PROBLEMS

Notes Handout ✓

Find the derivative of each function:

Ex. 1 $g(x) = \int_{\sin x}^{\cos x} \ln t\,dt$

$g'(x) = \ln(\cos x)\cdot(-\sin x) - \ln(\sin x)(\cos x)$

Ex. 2 $g(x) = \int_{x^3}^{e^x} \tan t\,dt$

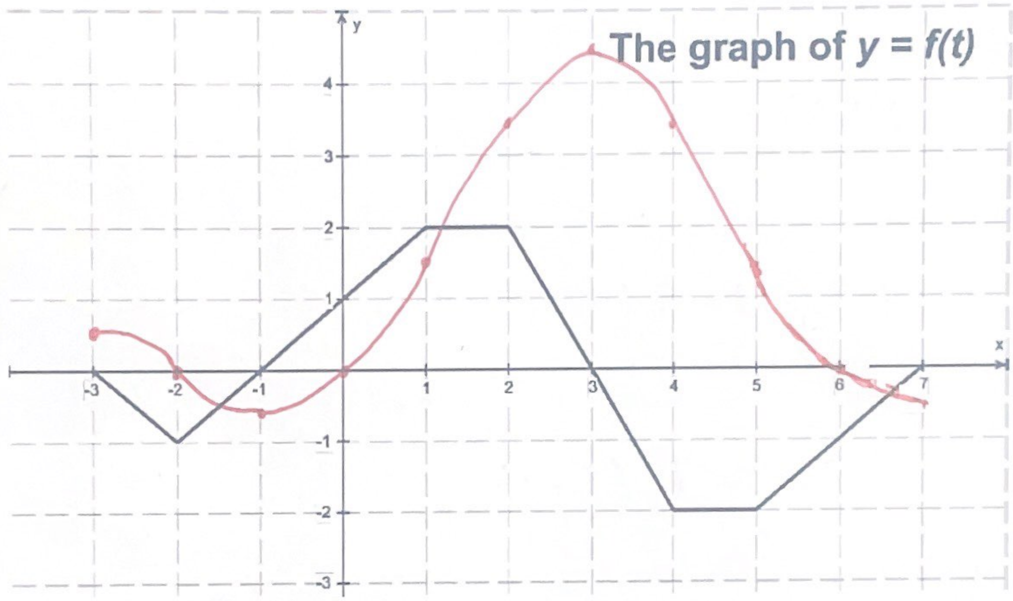
$g'(x) = \tan e^x \cdot (e^x) - 3x^2 \tan(x^3)$

HOMEWORK Worksheet 51

The Accumulation Function

Example 1

Let $g(x)=\int_0^x f(t)\,dt$. The graph of $y=f(t)$ is given below.



Fill in the values of the table. (Note: the lower limit is 0)

$g'(x)=f(x)$

$g''(x)=f'(x)$

x	$g(x)$	$g'(x)=f(x)$	$g''(x)$
-3	$\frac{1}{2}$		
-2	0	-1	und
-1	$-\frac{1}{2}$	0	1
0	0	1	1
1	$\frac{3}{2}$	2	und
2	$\frac{7}{2}$	2	und
3	$\frac{9}{2}$	0	-2
4	$\frac{7}{2}$	-2	und
5	$\frac{3}{2}$	-2	und
6	0	-1	1
7	$-\frac{1}{2}$		

Example 2

Use the values from your table to sketch a graph of $y = g(x)$. Be sure to indicate all relative extrema and points of inflection.

Use your graph as an aide to answer the following questions.

1. On what interval(s) is the graph of $y = g(x)$ increasing?

   $(-1, 3)$

2. On what interval(s) is the graph of $y = g(x)$ decreasing?

   $(-3, -1)$ & $(3, 7)$

3. On what interval(s) is the graph of $y = g(x)$ concave up?

   $(-2, 1)$ & $(5, 7)$

4. On what interval(s) is the graph of $y = g(x)$ concave down?

   $(-3, -2)$ & $(2, 4)$

5. How can you use the graph of f to determine where $y = g(x)$ has a local minimum?

   $g' = f$ Δs from $-$ to $+$

6. How can you use the graph of f to determine where $y = g(x)$ has a local maximum?

   $g' = f$ Δs from $+$ to $-$

7. How can you use the graph of f to determine where $y = g(x)$ has a point of inflection?

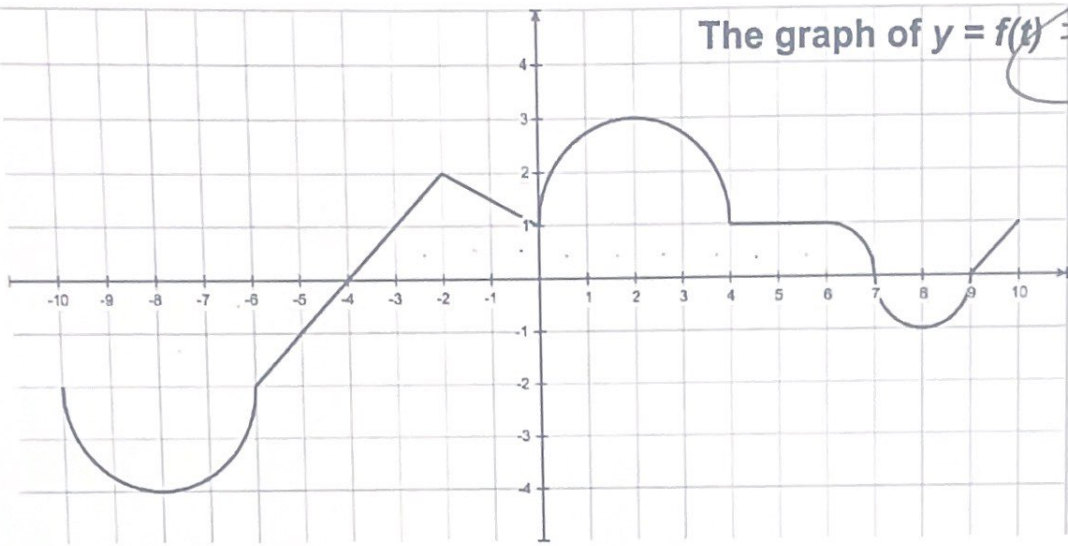
   $g'' = f'$ Δs signs

   f Δs from inc to dec or dec to inc

Example 3

Let $g(x)=\int_{-4}^{x} f(t)\,dt$. The graph of $y=f(t)$ is given below.

$g' = f(x)$



x	$g(x)$	$g'(x) = f(x)$	$g''(x) = f'$
−8	$6+\frac{1}{4}\pi(4)=6+\pi$	-4	0
4	$9+\frac{1}{2}\pi(4)=9+2\pi$	1	und
7	$9+2\pi+2+\frac{1}{4}\pi$	0	und

a. The graph of $y=g(x)$ is increasing on $(-4,7)$ $(9,10)$ because $g'>0$.

b. The graph of $y=g(x)$ is decreasing on $(-10,-4)$ $(7,9)$ because $g'<0$.

c. The graph of $y=g(x)$ is concave down on $(-10,-8)$ $(-2,0)$ $(2,4)$ $(6,8)$

because g' is decreasing $(g''<0)$.

d. The graph of $y=g(x)$ is concave up on $(-6,-2)$

because ______.

e. The graph of $y = g(x)$ has a local maximum at $X = 7$

because g' Δs from + to −.

f. The graph of $y = g(x)$ has a local minimum at $X = -4,\ X = 9$

because g' Δs from − to +.

g. The graph of $y = g(x)$ has a point of inflection at $X = -8, -2, 2, 8$

because g'' Δs signs.

Example 4 – Complete each statement

$g(x) = \int_0^x f(t)dt$ is increasing whenever the graph of f is:

$g(x) = \int_0^x f(t)dt$ is decreasing whenever the graph of f is:

$g(x) = \int_0^x f(t)dt$ is concave up whenever the graph of f is:

$g(x) = \int_0^x f(t)dt$ is concave down whenever the graph of f is:

$g(x) = \int_0^x f(t)dt$ has a local extrema whenever the graph of f:

$g(x) = \int_0^x f(t)dt$ has a point of inflection whenever the graph of f: